

# Bank runs and accounting for illiquid bank assets

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## Abstract

We investigate the role of mandatory fair-value versus historic-cost reporting in a bank. The bank has invested in long-term financial claims on the yield of specific real assets. It is financed by deposits that can be withdrawn on short notice. Withdrawals must be financed by fire sales on the secondary market where both depositors and the secondary market rely on the bank's accounting report regarding the value of the assets. Due to asset illiquidity, withdrawals are costly and always inefficient. In response to this tension, the bank under-invests. We show that a curbed information pattern, as generated by historic-cost accounting, mitigates under-investment compared to fair-value accounting. However, the bank favors fair-value accounting to historic cost accounting if asset illiquidity is not too severe. Finally, if depositors' ex ante beliefs are too skeptical, investment is impossible under historic-cost accounting and fair-value accounting is the only remaining method.

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# 1 Introduction

In this paper we investigate the common claim that fair value accounting has the potential to accelerate deposit withdrawals from sick banks and trigger bank runs. This claim has been particularly made in the 2008 financial crisis where fair value accounting was blamed to accelerate fire sales and weaken banks' stressed capital base. In an overnight action, the IASB amended the standard for accounting for financial instruments, IAS 39, and allowed for re-classifications of financial assets out of the fair-value categories into the historic-cost categories.

We ask how the accounting measurement of illiquid financial assets influences potential bank runs. The bank has invested in long-term financial claims on the yield of specific real assets. These assets are financed by deposits that can be withdrawn on short notice. Such withdrawals must be financed by fire sales on the secondary market where both depositors and the secondary market rely on the bank's accounting report regarding the value of the financial assets. Due to asset illiquidity, withdrawals are costly and always inefficient. In response to this tension, the bank under-invests. We show that a curbed information pattern as generated by historic-cost accounting mitigates under-investment compared to fair-value accounting.

We consider stylized versions of the two major accounting methods: historic cost accounting (HC) and fair value accounting (FV). Assuming that depositors base their withdrawal decisions exclusively on the accounting report ("the signal"), we investigate the equilibrium withdrawal patterns under either of the two accounting rules. We show that HC dominates FV from an ex post point of view (after the investment decision has been made), because the subset of signal realizations that triggers a run is partially pooled with

the subset that does not trigger a run. Given this result, there is a regulatory tension because the bank favors FV if asset liquidity is sufficiently high.

The key driver for our results is asset illiquidity at the withdrawal stage. Similar to Plantin, Sapra, and Shin (2008) we assume that the bank has superior abilities to detect investments into specific assets compared to the rest of the economy. For instance, think of specialization in innovative firms, or firms in the local economy. For such investments, the bank boosts the value of the underlying real asset by combining financing with specific advising. The basic tension in the model arises because such an exclusive expertise implies that the secondary market for these assets is imperfect. If the bank must liquidate (some of) its assets in order to finance withdrawals at the interim stage, the assets can only be sold with losses. In other words: the bank's financial assets are illiquid.<sup>1</sup>

Given that the bank carries illiquid assets, we first show that a FV bank is prone to bank runs. Consider a bank that contracts with a given number of depositors. Suppose the bank has two options, offering a contract with a high or a low nominal redemption amount. If it offers the high contract, depositors won't be aggressive at the withdrawal stage, resulting in low expected losses from withdrawals. However, the high redemption amount also leads to low expected profits for the bank. Alternatively, the bank might offer the low redemption amount. This would lead to a higher expected profit, but more aggressive withdrawal behavior. In summary, the fair value bank trades off withdrawal costs against expected profit.

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<sup>1</sup>As an example, suppose the bank has invested 100 million currency units (CU) in a financial assets at date 0. At date 1 the intrinsic asset value drops to CU80 million. Now suppose the depositors consider withdrawing their deposits. This is inefficient because the bank can sell these assets only for, say, CU70 million because outsiders have limited use of the asset. As a result, there is an additional loss of CU10 million.

Compare this to an HC bank. Under HC, there is no disclosure of the expected asset value at the interim date, unless the standard triggers an impairment. Therefore, all favorable realizations of the expected asset value are pooled. Thus, depositors behave as if they were linked to an average bank with favorable information (similarly to Göx & Wagenhofer 2009). For any given redemption value, the depositors either withdraw or stay after non-impairment. However, if depositors anticipate withdrawals under the low redemption amount, they would never sign the deposit contract in the first place. Therefore, the bank is forced to offer the contract with the high redemption amount. In other words, HC creates a commitment not to trade off expected bank profits against costly withdrawals.

Our paper differs from previous work on bank runs. There, runs are classified in "fundamental based" and "panic based" where a fundamental based run occurs if the liquidation value of the asset exceeds the continuation value (and vice versa for the panic based run). By construction, fundamental based runs are impossible in our paper. In our model the outside use of its (financial) assets is limited. Thus, any bank run is panic based.

In both theoretical modeling and reality potential bank run situations are characterized by multiple equilibria (Diamond and Dybvig 1983). Most analytical approaches rely on assumptions on higher order beliefs ("Global Games") as equilibrium refinement in order to come to point predictions.<sup>2</sup> We are less restrictive and characterize a broader set of equilibria. While missing point predictions seem unpleasant at first sight, we believe that this is a suitable approach to analyze bank runs. For instance, politicians are aware that there are many equilibria when they advise the population to

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<sup>2</sup>See for example Morris & Shin (2000) ; Goldstein & Pauzner (2005) or Gao & Jiang (2014).

"trust in the banks". We show that our central result, namely that HC may outperform FV, may reverse if the depositors are sufficiently pessimistic about other depositors' withdrawal behavior at the outset and therefore play a bad equilibrium rather than a good equilibrium.

Our comparison of FV versus HC accounting has links to the recent literature on conservatism in accounting. We use a similar setting as Gigler, Kanodia, Sapra, and Venugopalan (2009). These authors show that neutral accounting (FV) dominates conservative accounting (HC). We obtain the opposite result because withdrawal decisions cannot be contracted upon in our model and liquidation is never efficient.

Our paper complements the vast literature on transparency of the banking system, as reviewed in Goldstein and Sapra (2013). We extend the analysis of Plantin, Sapra, and Shin (2008) who show that inefficiencies are lower under FV accounting than under HC accounting if assets are sufficiently liquid, while the reverse holds for illiquid assets. Plantin, Sapra, and Shin work with unique equilibria and base the tradeoff between FV and HC on the bank manager's incentive to maximize earnings (control role of earnings). Our main result is derived from the informative role of earnings. It rests on the difference between the sets of multiple equilibria where it may happen that no deposit contract is feasible under HC, but exists under FV.

A related branch of the literature considers the publication of stress test results rather than accounting reports. Similar to our modeling of HC, this literature focusses on questions around the disclosure of bad information. Goldstein and Leitner (2015) show that a regulator should not disclose stress test results if the overall state of the industry is perceived as strong, while partial disclosure is optimal otherwise. Bouvard et al (2015) derive parallel

results, supplemented by an analysis of the regulator's incentives to publish the stress test results. Moreover, Prescott (2008) highlights the indirect effects of disclosure. He argues that public dissemination of information can hurt the ability to collect it in the first place.

Similarly to curbing information by using HC rather than FV, panic based runs may also be reduced by earnings management. Gao and Jiang (2014) show that under the threat of a run, managers with poor pre-manipulation earnings tend to manipulate the earnings report upward to the withdrawal threshold level of earnings, thus pooling themselves with better types.

The remainder of the paper is organized as follows. The next section introduces the model. Section 3 considers the bank-run equilibria under both FV and HC. Finally, we investigate the bank's upfront investment decision in section 4. A few conclusions follow in Section 5.

## 2 The model

A bank operates for three stages as shown in Figure 1. At *date 0* there are  $n$  symmetric depositors. Each depositor lends a (cash) amount of  $I$  to the bank. Still at date 0, the bank invests its entire capital,  $nI$ , in a financial asset. For example, think of investments into the shares of an innovative firm. Each invested money unit yields an uncertain cash return  $x$  at date 2. We assume that  $x$  has positive density  $f(x)$  over  $[0, \bar{x}]$  and  $E[x] > 1$ . For simplicity, normalize all discount factor to 1. Thus, the bank holds financial assets with an intrinsic value of  $E[x]nI$  at the end of date 0.

The deposit contract specifies a nominal redemption amount  $D$  that the bank is supposed to repay at date 2,  $D \geq I$ . The total face value of debt is

date 0	date 1	date 2
Deposit contracts written	Signal $y$ Bank reports $B(y)$	if bank didn't crash at date 1:
Cash inflow: $nI$ Nominal debt $nD$	Depositors decide whether to withdraw	Return on remaining capital realized
Bank invests $nI$	Bank liquidates assets to finance withdrawals	

Figure 1: Time line

$nD$ .

At *date 1* the bank privately observes a signal  $y \in [\underline{y}, \bar{y}]$  about  $x$ . The bank updates the expected return to  $E[x|y]$  using Bayes' rule. We denote the marginal density function of  $y$  by  $g(y)$  and the conditional density function of  $x$  by  $f(x|y)$ . Moreover, we assume that the monotone likelihood ratio property holds (Milgrom 1981). Therefore,  $E[x|y]$  is strictly increasing in  $y$ . Let  $\lim_{y \rightarrow \underline{y}} E[x | y] = 0$  and  $\lim_{y \rightarrow \bar{y}} E[x | y] = \bar{x}$ .

Date-1 information about  $y$  is mapped into the bank's book value  $B$  of financial assets at date 1. To specify this book value we consider two candidate rules, namely, Fair-Value Accounting (FV) and Historic-Cost Accounting (HC). For FV we assume that the book value is proportional to  $E[x|y]nI$ .<sup>3</sup> Under HC,  $B$  is the minimum of the historical investment amount  $nI$  and  $E[x|y]nI$ . This minimum rule applies because bank's assets must be impaired if  $E[x|y]$  drops below 1. We define the *impairment threshold*  $y_1$  by

$$E[x | y_1] = 1. \tag{1}$$

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<sup>3</sup>More precisely, the Fair Value is given by the exit price. In the language of IFRS 13, we have a level-2 investment.

We assume that the bank strictly follows these rules, i.e. it cannot bias the report.

In order to model bank runs we assume that each depositor has the opportunity to withdraw the deposit  $I$  at date 1, after  $B$  has been published. Since depositors do not observe  $y$ , they base their individual withdrawal decision on the published book value  $B$  after updating the density function to  $f(x|B)$ . We also assume that depositors have no access to any other signal than  $B$ .

Suppose that  $j \in \{0, \dots, n\}$  depositors withdraw a total amount of  $jI$  at date 1. The withdrawals must be financed by selling some of the bank's invested assets. Thus, the bank sells a share  $\kappa$  of its assets on the secondary market at the current market price  $P_1$ . To keep the model tractable we assume that both, depositors and the market, have no other information than the reported book value  $B$ . Therefore,  $P_1$  just depends on  $B$ . The necessary capital share  $\kappa$  is given by  $\kappa n I P_1 = jI$ . If positive, the *remaining capital base* is given by

$$K(j, B) = (1 - \kappa)nI = \left(n - \frac{j}{P_1}\right)I, \quad (2)$$

and the bank's cash flow at date 2 equals  $xK(j, B)$ . If the remaining capital base is negative, the bank crashes at date 1.

Liquidating assets at date 1 is costly because the market discounts the expected value of the assets by a factor  $\lambda < 1$ . The idea is that the bank is specialized in the regional economy, or the bank's employees might be specialists for some specific industry. Thus the real assets of that industry are *specific* in the sense that they have a greater value when the bank finances them rather than the anonymous capital market. If  $\lambda$  is low, specificity of

the underlying real assets is high and liquidity of the bank's financial assets is low. When the bank sells its assets to finance withdrawals, it destroys the extra value. This leads to a discount. Therefore, when the un-discounted value of an asset unit is  $E[x|B]I$ , the market price  $P_1$  is only  $P_1 = \lambda E[x|B]I$ . This assumption is crucial for what follows. Obviously, it is never efficient to liquidate the bank's assets at date 1. In the language of the previous literature (among others, see Morris & Shin (2000); Goldstein & Pauzner (2005); Gao & Jiang (2014)), any withdrawal is "panic based" in our model.

**Lemma 1** *Early liquidation of the bank's assets at date 1 is never efficient.*

To keep the basic model simple, we assume that the number of depositors  $n$  is "large" in the sense that<sup>4</sup>

$$\lambda \in \left( \frac{1}{n}, 1 \right).$$

Before we proceed to the analysis we quickly analyze what would happen if we prevented the depositors from withdrawing their deposits. Then, the First Best would be achieved.<sup>5</sup> Without withdrawals, the problem boils down to a standard debt contract. Denote the minimum rate of return such that the bank does not crash at date 2 by  $x_0 = \frac{D}{I}$ . The bank maximizes its expected date 2 payoff that is given by the expected excess net cash flow in the case of survival,

$$\int_{x_0}^{\bar{x}} n(xI - D)f(x)dx,$$

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<sup>4</sup>Otherwise results do not change fundamentally, but the analysis requires more case distinctions.

<sup>5</sup>Equivalently, the First Best is achieved if the bank was committed to be silent at date 1. If the bank were silent, depositors would receive no new information at date 1. Thus, they would not revise their participation decision in the first place. Goldstein and Pauzner (2005) show that the possibility of withdrawing may be an endogenous part of an optimal deposit contract.

under the depositors' participation constraint

$$\int_0^{x_0} xIf(x)dx + \int_{x_0}^{\bar{x}} Df(x) \geq I.$$

The bank chooses  $D$  as low as possible, making the participation constraint bind. Solving the binding constraint for  $D$  and substituting into the objective function shows that the bank obtains the entire net cash return from investment, as given by

$$E[x]nI - nI.$$

Thus, the bank invests if and only if  $E[x] \geq 1$ .

Based on this observation, there is a potential source of inefficiency, once the bank has no contractual means to prevent depositors from withdrawing. Expected losses from fire sales at date 1 will lead to under-investment in the first place, i.e. the bank invests if and only if  $E[x] > 1$ .

### 3 Accounting information based withdrawals

#### 3.1 Withdrawals based on fair value accounting

According to IFRS 13, the *fair value* of an asset is given by its *exit price*, i.e. the price that would be received when selling the asset. Therefore, the fair value of one unit of the bank's assets at date 1 is given by  $P_1 = n\lambda E[x|y]I = B^{FV}$ . Due to MLRP, depositors infer the true value of  $y$  from observing  $B^{FV}$ . Thus, there is symmetric information between the bank, the depositors, and the market after the publication of  $B^{FV}$ . We therefore suppress the variable  $B^{FV}$  and work with  $y$  directly.

To begin with backward induction, suppose that  $D$  is given. Consider the depositor's individual withdrawal decision at date 1 after each depositor has observed  $y$  via  $B^{FV}$ . Suppose that each depositor believes that  $j \leq n - 1$  other depositors will withdraw their deposits. The best response depends on the individual depositor's expected payoffs from withdrawing the deposit, denoted by  $W(j, y)$ , or not withdrawing the deposit, denoted as  $V(j, y)$ .

Consider  $W(j, y)$  first. If the depositor decides to withdraw, the total number of withdrawals is  $j + 1$ . Two situation may occur. First, the bank may be able to pay out the total amount of  $(j + 1)I$ , so each depositor gets  $I$ . Second, the liquidation proceeds may not be sufficient to cover the total withdrawals. Then, the bank crashes. In the absence of a deposit insurance, each depositor gets his share of the liquidation proceeds,  $\lambda E[x|y]I$ . The expected payoff from withdrawing at date 1,  $W(j, y)$ , therefore depends on both the realization of  $y$  and  $j$ . For each  $j$  there exists a critical realization  $y_j^{crash}$  where fire sales of the bank's assets are just sufficient to cover all withdrawal claims.  $y_j^{crash}$  is given by  $\lambda E[x|y_j^{crash}]nI = jI$ . Summing these considerations up, we get

$$W(j, y) = \begin{cases} I & \text{if } y \geq y_{j+1}^{crash} \\ \lambda E[x|y]I & \text{if } y < y_{j+1}^{crash}. \end{cases} \quad (3)$$

Now suppose instead that the depositor leaves the deposit in the bank. Again, two situation may emerge. First, the bank may crash at date 1 anyway because it already cannot cover the other depositors' withdrawals. In this case, the depositor obtains his liquidation share  $\lambda E[x|y]I$ . If the bank survives  $j$  withdrawals at date 1, the remaining capital base of the bank is  $K(j, y)$ , as defined in equation (2). The depositor's expected payoff from

leaving the money in the bank is given by

$$V^+(j, y) = \int_0^{x_j} x \frac{K(j, y)}{n - j} f(x|y) dx + [1 - F(x_j|y)]D,$$

where  $x_j$  is the critical date-2 cash return such that the bank is able to pay the depositors' claims at date 2,

$$x_j = \frac{(n - j)D}{K(j, y)}.$$

Combining these two cases,  $V(j, y)$  is given by

$$V(j, y) = \begin{cases} V^+(j, y) & \text{if } y \geq y_j^{crash} \\ \lambda E[x|y]I & \text{if } y < y_j^{crash}. \end{cases} \quad (4)$$

The value  $y_n^{crash}$  where withdrawals by *all* depositors just lead to a crash of the bank will be of particular interest for the subsequent analysis. We will refer to it as the *bankruptcy threshold*. It is given by  $\lambda E[x | y_n^{crash}]nI = nI$  or, more shortly,

$$E[x | y_n^{crash}] = \frac{1}{\lambda}. \quad (5)$$

Note that a higher asset liquidity pushes the bankruptcy threshold to lower values. If the asset is liquid ( $\lambda \rightarrow 1$ ), the bankruptcy threshold is identical to the impairment threshold  $y_1$ , as defined in (1). For illiquid assets, the bankruptcy threshold exceeds the impairment threshold. For  $\lambda$  sufficiently small we have  $y_n^{crash} = \bar{y}$ .

Before we proceed with the economic analysis we need to develop some preliminary results regarding  $V^+(j, y)$ . A crucial feature of deposit withdrawals is their drawback on the expected payoff of those who do not withdraw. This drawback is complex by nature. Suppose that  $j$  depositors decide

to withdraw the total amount of  $jI$  from a financially *healthy bank*, i.e. a bank that is able to pay out all  $n$  depositors at date 1. The healthy bank gets a stronger capital base after any withdrawal. This is because any withdrawing depositor only gets  $I < D$ . Therefore, the likelihood increases that the remaining depositors receive  $D$  at date 2. We conclude that  $V^+(j, y)$  is increasing in  $j$  when  $y > y_n^{crash}$ .

To the contrary, consider  $j$  withdrawing depositors in a *sick bank* that is not able to pay out all the  $n$  depositors. In this case, each additional withdrawal stresses the bank's capital base even more. As a result, there is a lower likelihood that the remaining depositors receive  $D$  at date 2. Thus,  $V^+(j, y)$  is decreasing in  $j$  when  $y < y_n^{crash}$ .

Finally, MLRP implies that  $V^+(j, y)$  is increasing in  $y$ . These results are summarized in the following Lemma.

**Lemma 2**

$$V^+(j, y) \text{ is } \begin{cases} \text{decreasing in } j \text{ for all } & y < y_n^{crash} \\ \text{independent of } j \text{ for } & y = y_n^{crash} \\ \text{increasing in } j \text{ for all } & y > y_n^{crash} \end{cases}$$

Moreover,  $V^+(j, y)$  is increasing and concave in  $y$ .

**Proof.** All proofs are in the appendix.

Before we describe the equilibrium structure, we will compare the (maximum) payoff from withdrawing,  $W(j, y)$ , to the payoff from staying,  $V(j, y)$ . These two values are equal at the *withdrawal threshold*  $y_j^{leave}$ , i.e.

$$W(j, y_j^{leave}) = V(j, y_j^{leave}). \tag{6}$$

Obviously, the individual decision to withdraw depends on  $y$  and on the number of other depositors that withdraw. For instance, if  $V(0, y) \geq W(0, y)$ , no depositor wants to leave, given that no other depositor leaves or  $y \geq y_0^{leave}$ . Likewise, if  $V(n-1, y) < W(n-1, y)$ , all depositors will leave given that all other depositors leave, or  $y < y_{n-1}^{leave}$ .

The relative position of the withdrawal thresholds  $y_0^{leave}$  and  $y_{n-1}^{leave}$ , and their position relative to the bankruptcy threshold,  $y_n^{crash}$ , are crucial. Figure 2 illustrates  $V^+(0, y)$  and  $W(0, y)$  for a given liquidity level. The intersection<sup>6</sup> with  $W(0, y) = I$  determines the withdrawal threshold,  $y_0^{leave}$ . Consider the position of  $y_0^{leave}$  relative to  $y_n^{crash}$ . To this end, note that  $W(j, y)$  is independent of  $D$ , but  $V^+(j, y)$  shifts upward if  $D$  increases. If  $D$  is low, depositors are aggressive at the withdrawal stage because the expected payoff from staying  $V^+(j, y)$  is low. Therefore, they withdraw even when the realization of  $y$  is fairly good. Thus,  $y_0^{leave}$  exceeds the bankruptcy threshold. This situation is graphed in the left-hand picture of Figure 2. Contrary, for a sufficiently high redemption amount  $D$ ,  $y_0^{leave}$  is smaller than  $y_n^{crash}$  (see the right-hand graph). Depending on the liquidity of assets,  $\lambda$ , there exists a critical level of the redemption amount, where the withdrawal threshold  $y_0^{leave}$  just coincides with the bankruptcy threshold  $y_n^{crash}$ . We denote this level by  $\hat{D}(\lambda)$ .

Using these definitions we derive the following preliminary result that allows us to characterize the strategic interaction at the withdrawal stage.

**Lemma 3** *For any given  $\lambda$ , define a critical redemption amount  $\hat{D}(\lambda)$  by the solution to  $y_0^{leave} = y_n^{crash}$ . This critical boundary  $\hat{D}(\lambda)$  is increasing in  $\lambda$ .*

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<sup>6</sup>Concavity of  $V^+$  implies that  $V^+(j, y) - W(j, y)$  may change its sign twice, as can be seen from the graph. We deal with this complication in the Appendix. In our notation,  $y_j^{leave}$  is the greater of the two intersections between  $V^+(j, y)$  and  $W(j, y)$  where  $W(j, y) = I$ .

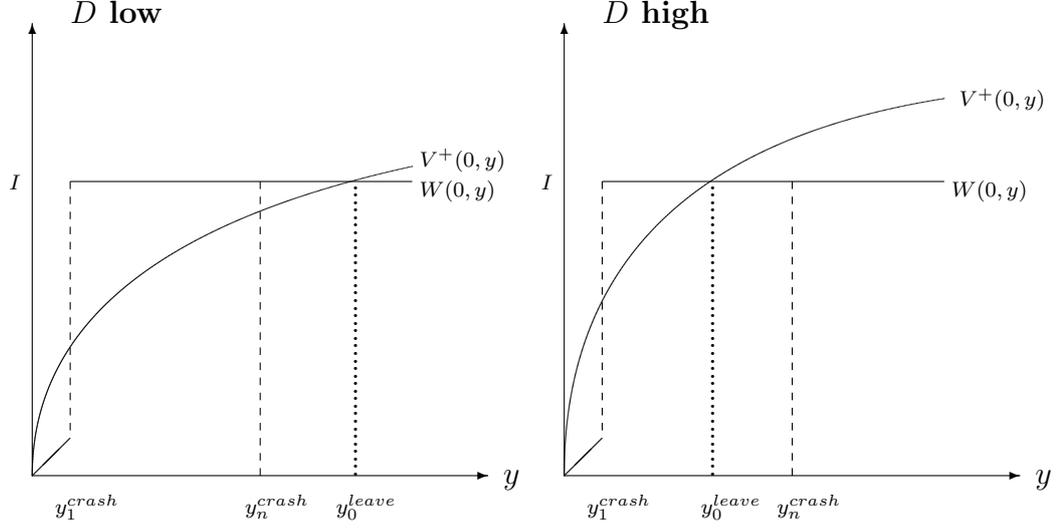


Figure 2: Relative position of  $y_n^{crash}$  and  $y_0^{leave}$

If  $D$  is low,  $D \leq \hat{D}(\lambda)$ , it holds that  $y_0^{leave} \geq y_{n-1}^{leave} \geq y_n^{crash}$  where equalities apply everywhere or nowhere.

Conversely, for  $D > \hat{D}(\lambda)$ , it holds that  $y_0^{leave} < y_n^{crash}$  and  $y_{n-1}^{leave} < y_n^{crash}$ . For  $n$  sufficiently large, it also holds that  $y_0^{leave} < y_{n-1}^{leave}$ .

Lemma 3 also sheds more light on the strategic interaction at date 1. Consider a bank that has to pay a low redemption amount,  $D < \hat{D}(\lambda)$ . In this case, the withdrawal activity starts for realizations  $y$  that even exceed  $y_n^{crash}$ . Thus, depositors leave the bank even though it is healthy. However, as soon as an additional depositor leaves a healthy bank, the remaining depositors are better off. This implies that withdrawals are strategic substitutes: Given

that sufficiently many other depositors leave, an individual depositor prefers to stay. Formally this implies that  $y_0^{leave}$  exceeds  $y_{n-1}^{leave}$ .

For banks with a high value of  $D$ , the situation is reverse. Withdrawals only occur in a sick bank. Then, the withdrawal decisions are strategic complements: Given that sufficiently many other depositors leave, an individual depositor prefers to leave too. The latter situation may cause "panic based runs". Formally,  $y_0^{leave}$  is lower than  $y_{n-1}^{leave}$ .

Now consider the equilibria. First, suppose again that  $D$  is lower than  $\hat{D}(\lambda)$ . Since the withdrawal interaction is in strategic substitutes, the withdrawal thresholds  $y_j^{leave}$  are decreasing in  $j$ . If  $y$  falls short of  $y_{n-1}^{leave}$ , the unique equilibrium action is that all depositors leave.<sup>7</sup> Moreover, if  $y$  is above  $y_0^{leave}$ , not withdrawing is the only equilibrium action. Between these two thresholds there exists a symmetric equilibrium withdrawal pattern in mixed strategies, where all depositors withdraw with some probability.

Second, suppose that  $D$  exceeds  $\hat{D}(\lambda)$ . Then, withdrawals are strategic complements. As a consequence, the withdrawal thresholds are piecewise increasing in the number of leaving depositors. Therefore, multiple equilibria may occur, as in Diamond and Dybvig (1983). To see this, notice first that no depositor leaving the bank if  $y$  is above  $y_0^{leave}$  constitutes one equilibrium. However, all depositors leaving if  $y$  is below  $y_{n-1}^{leave}$  is another. Since for  $n$  sufficiently large, we know that  $y_0^{leave} < y_{n-1}^{leave}$ , there are multiple equilibrium actions between the two thresholds. More precisely, there exists a continuum of symmetric pure strategy equilibria.

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<sup>7</sup>For very low  $y$ , i.e. if  $y$  goes to  $\underline{y}$ , there is also an area where not leaving constitutes another equilibrium. See the proof to Lemma 3.

**Proposition 1** *Suppose the bank reports under Fair Value accounting and  $\lambda$  is given. The set of equilibria at date 2 has the following properties.*

1. *For low redemption amounts,  $D < \hat{D}(\lambda)$ , a symmetric equilibrium strategy comprises*
  - *complete withdrawals if  $y$  falls short of  $y_{n-1}^{leave}$ ,*
  - *no withdrawals if  $y$  exceeds  $y_0^{leave}$ , and*
  - *randomized withdrawal strategies between  $y_{n-1}^{leave}$  and  $y_0^{leave}$ . The withdrawal probability  $p$  is decreasing in  $y$  with  $p = 1$  in  $y_{n-1}^{leave}$  and  $p = 0$  in  $y_0^{leave}$ .*
2. *For high  $D$ ,  $D > \hat{D}(\lambda)$ , and  $n$  sufficiently large, there are multiple equilibria where depositors withdraw if and only if  $y$  is in a subinterval  $(y_1^{crash}, \tilde{y})$  and do not withdraw for  $y > \tilde{y}$ . The cutoff level  $\tilde{y}$  can be any element of  $[y_{n-1}^{leave}, y_0^{leave}]$ .*

Figure 3 illustrates the result for sufficiently large  $n$ . The case of a high redemption amount with  $y_{n-1}^{leave}$  below  $y_0^{leave}$  is of special interest. Over the entire area between  $y_0^{leave}$  and  $y_{n-1}^{leave}$ , it may be that all depositors withdraw in equilibrium. It may also be that all depositors stay. Moreover, as the proof to the Proposition shows, there exists many other equilibria<sup>8</sup> where all withdraw if the realization  $y$  is below any threshold  $\tilde{y} \in (y_{n-1}^{leave}, y_0^{leave})$  and stay if  $y$  is above  $\tilde{y}$ .

The presence of multiple equilibria and the equilibrium selection problem are important. We confine ourself to rather mild refinements, i.e. we restrict

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<sup>8</sup>There also exists a mixed-strategy equilibrium where each depositor withdraws with probability  $p$ , with  $p$  declining in  $y$ .

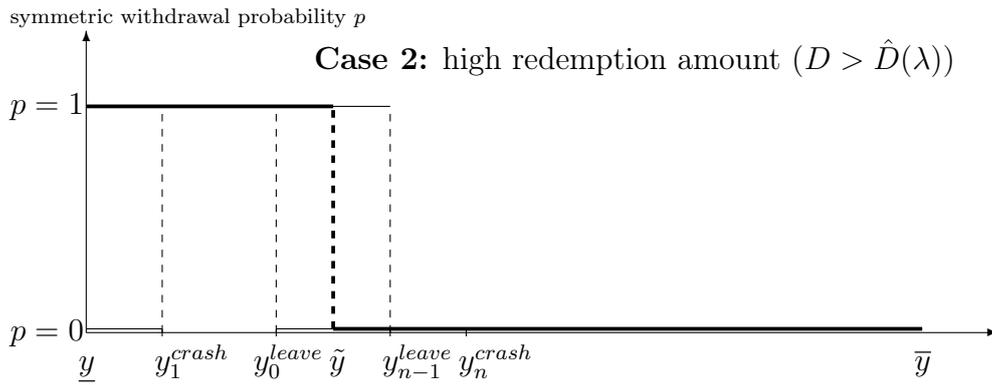
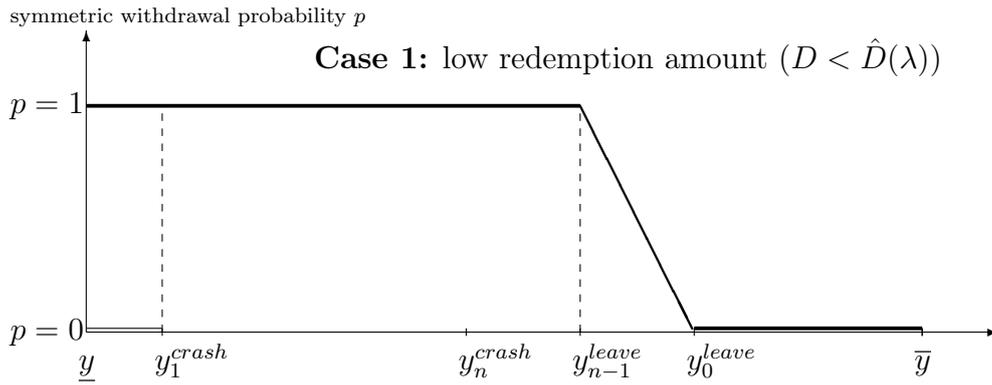


Figure 3: Stylized graphical illustration of Proposition 1 (for given  $D$ )

attention to symmetric equilibria with sequential rationality. Compared to the major body of the literature, we therefore consider the much larger set of equilibria, as described in Proposition 1.

### 3.2 Withdrawals based on historic cost accounting

Next, assume that the bank does Historic Cost accounting. Recall from the model description section that the impairment boundary,  $y_1$ , is given by  $E[x|y_1] = 1$ . If there is an impairment, the market learns the true signal realization  $y < y_1$ , and we are back to the case analyzed under FV.

Thus, HC differs from FV only in those cases where the realization of the signal is above  $y_1$ . Then, all realizations of  $y$  are pooled. When depositors observe no impairment, they update the expected rate of return to  $E[x|y \geq y_1] > E[x|y_1]$ . Thus, there is a discontinuity of the assessed value of the asset at the impairment threshold.

For each given number  $j$  of other depositors withdrawing, an individual depositor compares the expected continuation value,  $V(j, NI)$  to the expected withdrawal value,  $W(j, NI)$ , where  $NI$  stands for "no impairment". Similar to FV, the functions  $V(j, NI)$  and  $W(j, NI)$ , are defined piecewise, depending on whether the bank survives the withdrawals at stage 1 or not. The bank crashes if the withdrawal amount,  $jI$ , exceeds the maximum liquidation amount, i.e.  $jI > \lambda E[x|y \geq y_1]nI$  or

$$E[x|y \geq y_1] < \frac{j}{n} \frac{1}{\lambda}.$$

Consider  $W(j, NI)$  first. If an individual depositor withdraws, the total number of withdrawals is  $j + 1$ . If the bank crashes after  $j + 1$  withdrawals,

$W(j, NI)$  equals  $\lambda E[x \mid y \geq y_1]I$ . Otherwise it equals  $I$ .

$$W(j, NI) = \begin{cases} I & \text{if } E[x \mid y \geq y_1] \geq \frac{j+1}{n} \frac{1}{\lambda} \\ \lambda E[x \mid y \geq y_1]I & \text{if } E[x \mid y \geq y_1] < \frac{j+1}{n} \frac{1}{\lambda} \end{cases}$$

Similarly, we define  $V(j, NI)$  as

$$V(j, NI) = \begin{cases} \int_{y_1}^{\bar{y}} V^+(n-1, y)g(y \mid y \geq y_1)dy & \text{if } E[x \mid y \geq y_1] \geq \frac{j}{n} \frac{1}{\lambda} \\ \lambda E[x \mid y \geq y_1]I & \text{if } E[x \mid y \geq y_1] < \frac{j}{n} \frac{1}{\lambda}, \end{cases}$$

where we denote the upper part by  $V^+(j, NI)$ .

We prepare the analysis of the withdrawal stage by the following technical Lemma.

**Lemma 4** 1. For all  $\lambda < 1$  the impairment threshold is lower than the bankruptcy threshold,  $y_1 < y_n^{crash}$ .

2. For all  $D < \bar{x}I$ , we have  $y_1 < y_0^{leave}$ .

3. Suppose that the bank does not crash even if all depositors withdraw after no impairment (i.e.  $E[x \mid y \geq y_1] \geq \frac{1}{\lambda}$ ). Then,

$$V(0, NI) \leq \dots \leq V(n-1, NI)$$

with equality either everywhere or nowhere.

4. Suppose that the bank does not crash if  $j < n$  depositors withdraw, but crashes if  $j+1$  depositors withdraw, (i.e.  $\frac{j}{n\lambda} < E[x \mid y \geq y_1] < \frac{j+1}{n\lambda}$ ). Then,

$$V(0, NI) > \dots > V(j, NI).$$

The Lemma shows that the impairment threshold is always below the bankruptcy threshold. If  $y_1$  is realized, the total liquidation proceeds are less than  $nI$  since assets are illiquid. Thus, the bank will not survive a run after an impairment.<sup>9</sup>

Given that the bank does not survive a run, it is important to know whether the run occurs. Part 2 shows that the withdrawal threshold  $y_0^{leave}$  is above the impairment threshold. This implies that after any impairment, the equilibrium withdrawal actions are identical to those under FV. In particular, a run is part of an equilibrium outcome for all realizations of  $y$  below  $y_1$ . We conclude that any difference between FV and HC results from the different information structure after no impairment.

Parts 3 and 4 of the Lemma consider the no impairment case. The equilibrium action are no longer contingent on the actual  $y$ . Comparing  $E[x | y \geq y_1]$  and  $E[x | y_n^{crash}] = \frac{1}{\lambda}$ , we distinguish the cases where the bank survives full withdrawals after no impairment and where it crashes. Start with the presumption that the bank survives full withdrawals (Part 3). As argued before, each early withdrawal of  $I$  makes the remaining depositors better off because they get the full redemption amount  $D$  more likely. The more depositors withdraw, the better for the remaining depositors. Therefore,  $V(j|NI)$  is increasing in  $j$ .

Part 4 says that the result reverses if the bank does not survive a hypothetical full withdrawal. Knowing that the bank is not able to pay out all depositors, a lower number of withdrawals makes the remaining depositors better off. Therefore,  $V(j|NI)$  is decreasing in  $j$ .

Given these results we are finally able to derive the following Proposition

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<sup>9</sup>If the bank had sufficient equity, this would be different.

about the equilibrium play at the withdrawal stage. We show that a bank that did not impair survives a full withdrawal if the project is favorable in the sense that  $E[x|y \geq y_1] \geq \frac{1}{\lambda}$ . Equivalently, we may say that the investment project is quite liquid. In this case, the equilibrium behavior at the withdrawal stage depends on the redemption amount  $D$ . If  $D$  is sufficiently large, the unique equilibrium behavior is to stay. The cutoff value  $D_0$  is given by the solution to

$$V(0, NI) = I.$$

If  $D$  is lower, there is another cutoff value,  $D_{n-1}$ , with

$$V(n-1, NI) = I$$

such that for all  $D$  below  $D_{n-1}$  all depositors withdraw in equilibrium. For any  $D$  between these two cutoff levels, the equilibrium behavior is in mixed strategies.  $D_0$  exceeds  $D_{n-1}$  because withdrawals are strategic substitutes. Thus, equilibrium actions are unique.

Now consider the opposite case of a rather unfavorable or illiquid project where withdrawals are strategic complements, where  $E[x|y \geq y_1] < \frac{1}{\lambda}$ . Start with the case where  $\frac{1}{\lambda}$  just exceeds  $E[x|y \geq y_1]$ . In this case, the cutoff value  $D_0$  is smaller than  $D_{n-1}$  which yields multiple equilibria for any  $D$  between these two cutoff values. By definition, the two cutoff values are decreasing in asset liquidity  $\lambda$ . If  $\lambda$  is decreased even further,  $D_{n-1}$  will eventually hit the upper border of the return-on-investment interval,  $\bar{x}I$ . In words, asset liquidity has fallen that much that the bank can no longer pay out  $n-1$  withdrawing depositors. In this case, the all-withdrawal equilibrium exists for all  $D$ , but for  $D \geq D_0$  a non withdrawal equilibrium still co-exists.

**Proposition 2** *Suppose the bank reports under HC, and  $\lambda$  and  $D$  are given.*

1. For all  $(\lambda, D)$  a complete withdrawal an equilibrium after an impairment.
2. Consider a favorable (or liquid) project with  $E[x|y \geq y_1] > \frac{1}{\lambda}$  and the event of non-impairment. For any  $\lambda$  there exists some  $D_0 > I$  such that no depositor withdraws if  $D \geq D_0$ . There also exists a  $D_{n-1} < D_0$ , such that all depositors withdraw for all  $D < D_{n-1}$ . For  $D$  between  $D_{n-1}$  and  $D_0$  there is a mixed strategy equilibrium where each depositor withdraws with probability  $p(\lambda, D)$ .
3. Consider an unfavorable (or illiquid) project with  $E[x|y \geq y_1] < \frac{1}{\lambda}$  and non-impairment. If  $D \geq D_0$ , there exists an equilibrium where no depositor withdraws. If the bank survives  $n - 1$  withdrawals, there is also an equilibrium where all depositors withdraw for any value of  $D < D_{n-1}$ . If the bank does not survive  $n - 1$  withdrawals, the full withdrawal equilibrium exists for any  $D$ . Additionally, mixed strategy equilibria exist where each depositor withdraws with probability  $p(\lambda, D)$ .

In summary, the Proposition describes a similar equilibrium structure as under FV. The difference is that the withdrawal behavior is independent of  $y$  once there was no impairment. For this case, the equilibrium behavior is unique if the project is favorable (or liquid) because the no withdrawal cutoff amount  $D_0$  is above the full withdrawal amount  $D_{n-1}$ . For high  $D$  there is a complete withdrawal, for low  $D$  there is no withdrawal and for intermediate  $D$ s there are mixed withdrawals. Of course, for what follows, we will be interested in those  $D$ s where there are no withdrawals because this is a necessary condition for the depositors participating in the first place. We conclude that the bank must offer at least  $D \geq D_0$ .

With unfavorable (illiquid) projects there are again multiple equilibria after no withdrawal because the no withdrawal cutoff amount  $D_0$  is below the full withdrawal amount  $D_{n-1}$ . Again, a necessary condition for gathering the deposits is that the bank offers  $D \geq D_0$ . As we work out in the next subsection, depositors' participation is not guaranteed, though, because if depositors believe that the no withdrawal equilibrium won't be played, they still wouldn't participate in the first place.

## 4 The Investment Stage

At date 0 the bank suggest a redemption value  $D$  such that depositors are willing to invest their money in the bank, given that they anticipate the behavior described in the previous section. For the ease of exposition let us continue the analysis of the HC case first before we return to the FV case later.

### 4.1 Investment under HC

Anticipating the result of Proposition 2 above, bank and potential depositors face one of two possible situations. In the first, the potential investment project is favorable (or liquid) in the sense that

$$E[x|y \geq y_1] \geq \frac{1}{\lambda}.$$

From Proposition 2 we know that depositors will stay after no impairment and leave after an impairment, provided that  $D$  exceeds some lower bound  $D_0$ . In the next Lemma we show that the depositors' participation constraints require a redemption amount  $D_{HC}$  that strictly exceeds  $D_0$ . Whenever the bank bids  $D_{HC}$ , the incentive constraint is slack and can be neglected.

Now consider

$$E[x|y \geq y_1] < \frac{1}{\lambda}.$$

Proposition 2 shows that there exist multiple equilibria at the withdrawal stage. Still we can find some  $D_0$  such that no depositor withdraws after non-impairment is one of the equilibria. For a moment, take this behavior for granted. Again we can show that the participation constraint requires a greater  $D_{HC}$ . Contrary, suppose that depositors believe in (mixed- or pure-strategy) withdrawals after no impairment. For those two belief systems, the participation constraint cannot be met because there is no state of nature where the depositors get more than  $I$ , but there are many states where they get less than  $I$ . We conclude that whenever we observe a contract at date 0, the depositors must believe that everybody stays after non-impairment.

The observation that no withdrawals after non-impairment are only a *necessary* condition for the participation constraint to be met implies that the participation constraint is tighter. Thus, we may neglect the no-withdrawal condition in the subsequent analysis and just work with the participation constraint that is given by

$$\int_{\underline{y}}^{y_1} \lambda I E[x|y] g(y) dy + \int_{y_1}^{\bar{y}} V^+(0; y) g(y) dy \geq I. \quad (7)$$

**Lemma 5** *Under HC, the bank will invest only if depositors believe that the deposits are not withdrawn after non-impairment. Then the participation constraint is binding.*

So far we may summarize that the bank offers a redemption amount  $D_{HC}$  that leads to a binding participation constraint, given that the depositors

believe that there are no withdrawals after non-impairment. The amount  $D_{HC}$  follows from the binding participation constraint which implies that  $D_{HC}$  is decreasing in  $\lambda$ . If  $\lambda$  is close to 1, depositors can be attracted with a low redemption amount because withdrawal after an impairment lead to quite low expected dead-weight losses. Moreover, the equilibrium is unique because withdrawals are strategic substitutes. The more interesting case is that  $\lambda$  is small. For decreasing  $\lambda$  the minimum redemption value  $D_{HC}$  increases. Besides the direct effect, there is a strategic effect since withdrawals are strategic complements now. Each withdrawal erodes the bank's financial basis which implies that depositors must be compensated by an even higher  $D$ . For decreasing  $\lambda$  this argument can be continued until  $D_{HC}$  becomes so large that there expected bank surplus is completely eroded. In the model context this is the case if  $D_{HC} = \bar{x}I$ . If the expected surplus is eroded, the bank will no offer consider attracting deposits. Setting  $D_{HC}$  equal to  $\bar{x}I$  we obtain

**Proposition 3** *If the bank reports under HC, investment may take place only if*

$$\lambda \geq \lambda^{HC} = 1 - \frac{E[x] - 1}{E[x|y \leq y_1]G(y_1)} \quad (8)$$

The Proposition describes the minimum liquidity level that is necessary for a deposit contract to exist under HC.

## 4.2 Investment under FV

Our last step is to return to the FV case. From Proposition 1 we know that the game at the withdrawal stage may be in strategic complements or

substitutes, depending on whether the redemption amount  $D$  exceeds or falls short of an increasing borderline function  $\hat{D}(\lambda)$  (see Lemma 2).

Suppose for a while that the optimal contract was to agree on some high  $D$  such that we have strategic complements,  $D > \hat{D}(\lambda)$ . In a symmetric equilibrium all depositors withdraw if  $y < \tilde{y} \in (y_0^{leave}; y_{n-1}^{leave})$  and do not withdraw otherwise. This strongly resembles the previous situation under HC, with the difference that any of the possible cutoff values  $\tilde{y}$  exceeds the impairment threshold  $y_1$ . Borrowing from this similarity, we again obtain that the depositors' participation constraints bind and the incentive constraints can be ignored. Again,  $D$  is determined from the binding participation constraint and is decreasing in  $\lambda$ . Therefore, consistent to the initial presumption, we obtain high values of  $D$  for low degrees of asset liquidity  $\lambda$ . Denote the resulting minimum redemption value by  $D_{FV}$ . Of course,  $D_{FV}$  is not unique but depends on the depositors' initial beliefs about the cutoff withdrawal signal  $\tilde{y}$ .

Now suppose that the optimal contract was to agree on a low redemption amount  $D < \hat{D}(\lambda)$  that yields strategic substitutes. Now we have three different behavioral patterns: no withdrawals for high realizations of  $y$ ,  $y > y_0^{leave}$ , mixed strategies for intermediate realizations, and full withdrawals for low realizations,  $y < y_{n-1}^{leave}$ . The mechanics of the model do not change fundamentally. Incentive constraints are satisfied by construction. Thus, the participation constraint is the eventually binding restriction to the profit maximization problem of the bank. Again,  $D$  is chosen such that the participation constraint binds with  $D$  decreasing in the degree of asset liquidity,  $\lambda$ . Consistent to the initial assumption (i.e.  $D < \hat{D}(\lambda)$ ) we obtain a low value of  $D$  for a high value of  $\lambda$ .

In summary, it is sufficient to consider binding participation constraints in both cases.

**Lemma 6** *If the bank invests under FV, the redemption value  $D_{FV}$  follows from the depositors' binding participation constraints .*

In the light of this technical finding, the second main result of the paper is quickly established. Since  $D$  is decreasing in  $\lambda$ , the bank's expected profit is exhausted for low values of asset liquidity (low values of  $\lambda$ ). Setting  $D = \bar{x}I$  immediately leads to the following Proposition.

**Proposition 4** *If the bank reports under FV, investment will take place only if*

$$\lambda \geq \lambda^{FV} = 1 - \frac{E[x] - 1}{E[x|y \leq \tilde{y}]G(\tilde{y})}. \quad (9)$$

### 4.3 Comparing FV and HC

A quick comparison of our four Propositions suggests that HC dominates FV. First, from Propositions 1 and 2, withdrawals occur in less states of nature under HC than under FV for each given  $(\lambda, D)$ . Second, the cutoff values for the liquidity level  $\lambda$  derived in Propositions 3 and 4 suggest that the investment set under FV is smaller than under HC. Notice however that these cutoff values have been derived under the assumption that depositors participate under HC which is only one of the possible equilibria at the contracting stage.

Let us postpone the multiple equilibrium problem for a while and consider the optimal contract choice conditional on depositors being sufficiently

optimistic at the contracting stage. To start with, consider the case of a very liquid project ( $\lambda \rightarrow 1$ ) first. Depositors know that liquidation is easy and the bank survives for all  $y \geq y_1 = y_n^{crash}$ . Thus, if depositors were to know  $y$ , any withdrawing depositor would not risk a haircut. Under the HC system, information about the precise value of  $y$  is withheld. Thus, situations with a true  $y$  close to  $y_1$  are pooled with better situations and depositors stay. Conditional on the realization  $y$  this is costly to the depositors. As a compensation, they are offered a higher redemption amount  $D$ . We conclude that for sufficiently high liquidity levels, the redemption amount under HC is higher than under FV.

The situation changes if  $\lambda$  becomes small. Now, each depositor knows that each single withdrawal has a negative externality. Around the impairment threshold all depositors would be better off by leaving their money in the bank, but given that all others withdraw, it is optimal for any individual depositor to withdraw too. Therefore, HC depositors benefit from the pooling with favorable  $y$ 's. Since cases where the true value of  $y$  is just above  $y_1$  are lumped together with much better situations, depositors are committed not to withdraw. Contrary, FV depositors will still withdraw as long as  $y$  falls short of  $\tilde{y}$ . Therefore, if  $\lambda$  is sufficiently low, the necessary redemption amount under HC becomes smaller than under FV.

**Proposition 5** *For high asset liquidity, we have  $D_{HC} > D_{FV}$ , while low asset liquidity implies  $D_{HC} < D_{FV}$ .*

The Proposition gives a trace towards which accounting method is preferred by the bank. Trivially, the bank prefers HC if the level of asset liquidity is low. The opposite result for the high liquidity case is not that easy to see.

The bank will profit from a lower redemption amount  $D$ , but this comes at the cost of more withdrawals and higher expected efficiency losses from withdrawals. Nevertheless, the early withdrawal amount is  $I$  and any early withdrawal precludes the later payouts of the higher amount  $D$ . Since the bank survives the early withdrawals, the efficiency loss is outweighed by the other effects.

**Proposition 6** *For low asset liquidity banks prefer the HC method while for sufficiently high asset liquidity they prefer the FV method.*

The implication of this testable result is that banks prefer the less efficient accounting method if the level of liquidity is high. Thus, there is a potential reason for accounting regulators to intervene. It should be mentioned, though, that the resulting losses under a high liquidity level tend to be small. So depending on the cost of regulation, the regulator might also want to waive intervention.

We deliberately derived our results for entire sets of equilibria rather than point predictions. To this end, we only used mild refinements. A strength of this approach is that it allows to generate a prediction on the dependence of results on alternative belief systems that might prevail in the economy. For instance, as already mentioned, a direct implication of Propositions 3 and 4 is that HC strictly dominates FV if depositors are very optimistic regarding the other depositors' withdrawal behavior such that they participate under HC.

**Corollary 1** *Suppose that depositors are optimistic regarding the other depositors' withdrawal behavior. Then, HC dominates FV. In particular, if*

$\lambda \in (\lambda^{HC}; \lambda^{FV})$ , no contract exists under FV, but a contract is feasible under HC.

However, we might also assume that beliefs are pessimistic in the sense that each depositor assumes that the other depositors panick easily. Then no contract is feasible under HC whereas depositors know that no withdrawals can be part of a sequential equilibrium if  $y \geq y_{n-1}^{leave}$  under FV.

**Corollary 2** *Suppose that depositors are maximum pessimistic regarding the other depositors' withdrawal behavior. Then, there exists no deposit contract under HC whereas an FV contract is feasible for all*

$$\lambda \geq \lambda^{FV} = 1 - \frac{E[x] - 1}{E[x|y \leq y_{n-1}^{leave}]G(y_{n-1}^{leave})}.$$

The last Corollary appears to be based on sound intuition. If the economy is pessimistic, a flexible accounting / withdrawal mechanism attracts depositors at least in those cases where depositors can benefit from the flexibility. Thus, it is possible to satisfy even pessimistic depositors' participation constraint. More rigid information provision destroys the flexibility and leads to a credit crunch in presence of skeptical depositors.

## 5 Conclusion

For any given asset liquidity level and any given redemption value of bank deposits historic cost accounting (HC) curbs costly deposit withdrawals relative to fair value accounting (FV), because favorable information is pooled. This result holds for the entire set of possible equilibria, given that depositors

participate in the first place. Moreover, we have shown that the investment set under FV is a subset of the investment set under HC if depositors are sufficiently optimistic that other depositors will stay at the withdrawal stage. Taking both results together, HC dominates FV. However, HC is detrimental regarding the feasibility of deposit contracts if the initial beliefs regarding what happens at the withdrawal stage are sufficiently pessimistic. Then, FV dominates HC.

Our results provide a rationalization of the IASBs move to allow for reclassifications from FV to HC at the peak of the 2008 financial crisis. According to our model this was an effective measure to curb withdrawals, once the deposit contracts had been written. On the other hand, given the overall pessimism in the economy at that time, attracting new investments might have been more difficult for those banks that heavily reclassified.

Finally we show that there is a potential mismatch between individual preferences of banks and efficiency. Even in situations where depositors are not pessimistic and willing to lend money to the bank under both accounting methods, the bank prefers FV if the liquidity of its assets is sufficiently high. In this case, the bank prefers the less efficient accounting method.

For tractability reasons we assumed a no-equity bank. Obviously, imposing minimum equity requirements upon the bank will relax the pressure from panic based bank runs. That is, in a bank with sufficient equity, depositors will tolerate adverse releases of the interim accounting report more easily. Lu, Sapra, and Subramanian (2014) show that capital requirements should be counter-cyclical. Stricter capital requirement mitigate asset substitution, but exacerbate debt overhang.

## Appendix

### Proof of Lemma 2

Replace  $V^+(j, y)$  by  $\hat{V}(r, y)$  where  $r$  is a real number.

$$\begin{aligned} \frac{\partial \hat{V}(r, y)}{\partial r} &= \frac{1}{(n-r)^2} \int_0^{x_r} \left( nI - \frac{rI}{\lambda E[x|y]} \right) x f(x|y) dx - \frac{1}{n-r} \int_0^{x_r} \frac{I}{\lambda E[x|y]} x f(x|y) dx \\ &\quad + \underbrace{\left[ \frac{1}{n-r} \left( nI - \frac{rI}{\lambda E[x|y]} \right) x_r - D \right]}_{=0} f(x_r|y) \frac{\partial x_r}{\partial r} \\ &= \frac{1}{(n-r)^2} \int_0^{x_r} \left( nI - \frac{nI}{\lambda E[x|y]} \right) x f(x|y) dx, \end{aligned}$$

which implies that

$$\text{sign} \left( \frac{\partial \hat{V}(r, y)}{\partial r} \right) = \text{sign} \left( nI - \frac{nI}{\lambda E[x|y]} \right).$$

Using  $y_n^{crash}$ , we get

$$\frac{\partial \hat{V}(r, y)}{\partial r} \begin{cases} > 0 & \text{if } y > y_n^{crash}, \\ < 0 & \text{if } y < y_n^{crash}. \end{cases}$$

Similarly, we have

$$\begin{aligned} \frac{\partial V^+(j, y)}{\partial y} &= \int_0^{x_j} \frac{x}{n-j} \left( nI - \frac{jI}{\lambda E[x|y]} \right) \frac{\partial f(x|y)}{\partial y} dx + D \int_{x_j}^{\bar{x}} \frac{\partial f(x|y)}{\partial y} dx \\ &\quad + \int_0^{x_j} \frac{x}{(n-j)} \frac{jI}{\lambda E[x|y]^2} \frac{\partial E[x|y]}{\partial y} f(x|y) dx \\ &\quad + \underbrace{\left[ \left( nI - \frac{jI}{\lambda E[x|y]} \right) \frac{x_j}{n-j} - D \right]}_{=0} f(x_j|y) \frac{\partial x_j}{\partial y} \\ &= \int_0^{x_j} \left[ \frac{x}{n-j} \left( nI - \frac{jI}{\lambda E[x|y]} \right) - D \right] \frac{\partial f(x|y)}{\partial y} dx \\ &\quad + D \underbrace{\int_0^{\bar{x}} \frac{\partial f(x|y)}{\partial y} dx}_{=0} + \int_0^{x_j} \frac{x}{(n-j)} \frac{jI}{\lambda E[x|y]^2} \frac{\partial E[x|y]}{\partial y} f(x|y) dx. \end{aligned}$$

Partial integration reveals

$$\begin{aligned} \frac{\partial V^+(j, y)}{\partial y} &= -\frac{1}{(n-j)} \underbrace{\left( nI - \frac{jI}{\lambda E[x|y]} \right)}_{>0} \underbrace{\int_0^{x_j} \frac{\partial F(x|y)}{\partial y} dx}_{<0} \\ &\quad + \underbrace{\int_0^{x_j} \frac{x}{(n-j)} \frac{jI}{\lambda E[x|y]^2} \frac{\partial E[x|y]}{\partial y} f(x|y) dx}_{>0} > 0. \end{aligned}$$

Moreover, we get

$$\begin{aligned} \frac{\partial^2 V^+(j, y)}{\partial y^2} &= \int_0^{x_j} \underbrace{\left[ \frac{x}{n-j} \left( nI - \frac{jI}{\lambda E[x|y]} \right) - D \right]}_{<0} \underbrace{\frac{\partial^2 f(x|y)}{\partial y^2}}_{>0} dx \\ &\quad + 2 \frac{1}{(n-j)} \frac{jI}{\lambda E[x|y]^2} \frac{\partial E[x|y]}{\partial y} \underbrace{\int_0^{x_j} x \frac{\partial f(x|y)}{\partial y} dx}_{<0} \\ &\quad + \frac{1}{(n-j)} \frac{jI}{\lambda E[x|y]^2} \underbrace{\left( \underbrace{\frac{\partial^2 E[x|y]}{\partial y^2}}_{<0} - \frac{2}{E[x|y]} \left( \frac{\partial E[x|y]}{\partial y} \right)^2 \right)}_{<0} \int_0^{x_j} x f(x|y) dx \\ &\quad + \underbrace{\frac{\partial x_j}{\partial y}}_{<0} \frac{x_j}{(n-j)} \frac{jI}{\lambda E[x|y]} \frac{\partial E[x|y]}{\partial y} f(x_j|y) < 0. \end{aligned}$$

### Proof of Lemma 3

To define the critical boundary  $\hat{D}(\lambda)$ , note first that  $y_n^{crash}$  is independent of  $D$ , but depends on  $\lambda$  via  $E[x|y_n^{crash}] = \frac{1}{\lambda}$ . From  $1 > \lambda$  we get  $\bar{y} \geq y_n^{crash} > y_1$ . Second,

$$I = V^+(0; \hat{y}_0^{leave}) = \int_0^{x_0} Ix f(x|\hat{y}_0^{leave}) dx + D[1 - F(x_0|\hat{y}_0^{leave})] \quad (10)$$

defines  $\hat{y}_0^{leave}$  for each value of  $D$ . This function does not depend on  $\lambda$  directly. Since  $V^+(\cdot)$  is increasing in  $D$  and  $y$ ,  $\hat{y}_0^{leave}$  is decreasing in  $D$ . Since  $y_n^{crash}$  is decreasing in  $\lambda$  and  $\hat{y}_0^{leave}$  is decreasing in  $D$ ,  $\hat{D}(\lambda)$  is increasing in  $\lambda$  with  $D(\lambda = 0) = I$  and  $D(\lambda = 1) = \bar{x}I$ .

Suppose  $D = \hat{D}(\lambda)$  such that  $\hat{y}_0^{leave} = y_n^{crash}$ . Since  $V^+(j, y)$  is constant in  $j$  at  $y_n^{crash}$ , we have

$V^+(n-1; y_n^{crash}) = V^+(n-2; y_n^{crash}) = \dots = V^+(0; y_n^{crash}) = I$ . Reduce  $D$  to  $D < \hat{D}(\lambda)$  which implies  $V^+(j, y_n^{crash}) < I$  since  $V^+(j, y)$  is increasing in  $D$ . Therefore, for  $V^+(j, \hat{y}_j^{leave}) = I$  to hold we need to have  $\hat{y}_j^{leave} > y_n^{crash}$ . Since  $V^+(0; y)$  is increasing in  $j$  for  $y > y_n^{crash}$  we get  $y_n^{crash} < \hat{y}_{n-1}^{leave} < \hat{y}_{n-2}^{leave} < \dots < \hat{y}_0^{leave}$ .

For  $D > \hat{D}(\lambda)$  we get  $V^+(j, y_n^{crash}) > I$  such that  $\hat{y}_j^{leave} < y_n^{crash}$ . The arguments given above just reverse. Since  $V^+(0; y)$  is decreasing in  $j$  for  $y < y_n^{crash}$  we get  $y_n^{crash} > \hat{y}_{n-1}^{leave} > \hat{y}_{n-2}^{leave} > \dots > \hat{y}_0^{leave}$ .

For a complete characterization, we need to consider what happens in cases of low  $y$  where the bank cannot fully pay out all withdrawals, and depositors only get the bankruptcy share  $\lambda E[x|y]I$  instead. Define  $\tilde{y}_j^{leave}$  via

$$\begin{aligned} \lambda E[x|\tilde{y}_j^{leave}]I &= V^+(j; \tilde{y}_j^{leave}) \\ &= \frac{1}{n-j} \int_0^{x_j} \left( nI - \frac{jI}{\lambda E[x|y]} \right) x f(x|\tilde{y}_j^{leave}) dx + D[1 - F(x_j|\tilde{y}_j^{leave})] \end{aligned}$$

for  $\tilde{y}_j^{leave} < y_n^{crash}$ . Note that such a value does not need to exist. Suppose it does. Since  $V^+(\cdot)$  is increasing in  $y$ , we have  $\tilde{y}_j^{leave} < \hat{y}_j^{leave}$ . If  $D < \hat{D}(\lambda)$ , since  $V^+(0; y_n^{crash}) = \dots = V^+(n-1; y_n^{crash}) < I$  these borders may exist for some small values of  $j$  only. If  $D > \hat{D}(\lambda)$ , we have  $V^+(0; y_n^{crash}) = \dots = V^+(n-1; y_n^{crash}) > I$  such that these critical values always exist for all  $j$ .

Here, a complex structure may emerge if and only if  $y_j^{crash} < \tilde{y}_j^{leave} < y_{j+1}^{crash}$ . We then have

For  $y < y_j^{crash}$ ,  $V(j, y) = W(j, y) = \lambda IE[x|y]$  (indifferent),

for  $y_j^{crash} < y < \tilde{y}_j^{leave}$ ,  $V(j, y) = V^+(j, y) < W(j, y) = \lambda IE[x|y]$  (withdraw),

for  $\tilde{y}_j^{leave} < y < y_{j+1}^{crash}$ ,  $V(j, y) = V^+(j, y) > W(j, y) = \lambda IE[x|y]$  (leave)

for  $y_{j+1}^{crash} < y < \tilde{y}_j^{leave}$ ,  $V(j, y) = V^+(j, y) < W(j, y) = I$  (withdraw) and

for  $\tilde{y}_j^{leave} < y$ ,  $V(j, y) = V^+(j, y) > W(j, y) = I$  (leave).

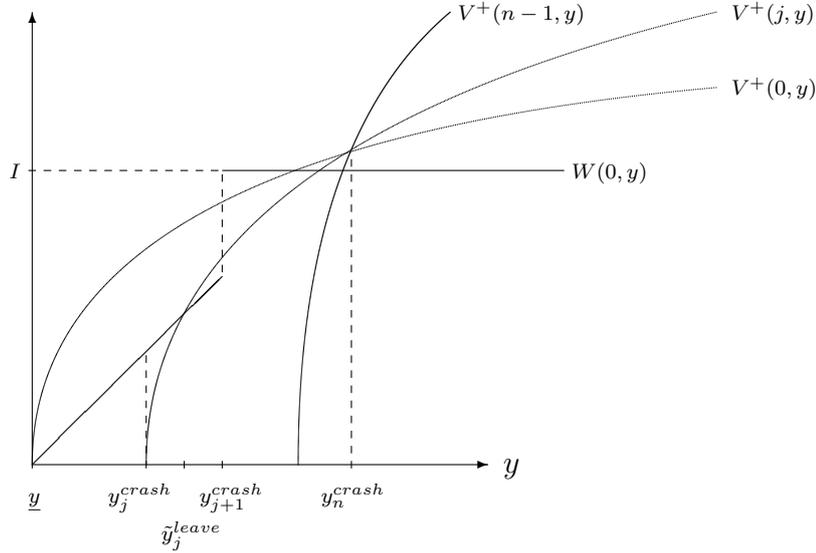


Figure 4: Multiple equilibria case,  $D > \hat{D}(\lambda)$

As our final step, we show that  $\hat{y}_0^{leave} < \tilde{y}_{n-1}^{leave}$  for  $D > \hat{D}(\lambda)$  and  $n$  sufficiently large. Take some arbitrary value of  $D$  with  $D > \hat{D}(\lambda)$ .  $\hat{y}_0^{leave}$  is independent of  $n$ . At the same time,  $V^+(n-1; y)$  starts at zero at  $y_{n-1}^{crash}$  and increases to  $V^+(n-1; y_n^{crash}) > I$  in  $y_n^{crash}$ , such that  $\tilde{y}_{n-1}^{leave} \in (y_{n-1}^{crash}, y_n^{crash})$ . But since  $E[x|y_{n-1}^{crash}] = \frac{n-1}{n} \frac{1}{\lambda} = \frac{n-1}{n} E[x|y_n^{crash}]$ , the interval  $(y_{n-1}^{crash}, y_n^{crash})$  shrinks towards  $y_n^{crash}$  from below as  $n$  increases. Since  $\hat{y}_0^{leave} < y_n^{crash}$  anyway,

and  $y_{n-1}^{crash}$  approaching  $y_n^{crash}$  arbitrary closely, we get  $\hat{y}_0^{leave} < \tilde{y}_{n-1}^{leave}$ .

To simplify the arguments hereafter, define

$$y_j^{leave} = \begin{cases} \tilde{y}_j^{leave} & \text{if } y_j^{crash} < \tilde{y}_j^{leave} < \hat{y}_j^{leave} < y_{j+1}^{crash} \\ \hat{y}_j^{leave} & \text{otherwise.} \end{cases} \quad (11)$$

### Proof of Proposition 1

Step 1: We aim to show the following is an equilibrium structure if  $D < \hat{D}(\lambda)$ .

*All depositors withdraw if  $y \leq y_{n-1}^{leave}$  which crashes the bank for  $y < y_n^{crash}$ .*

*Depositors withdraw with some probability  $p$  if  $y \in (y_{n-1}^{leave}, y_0^{leave}]$ .*

*All depositors leave their money if  $y > y_0^{leave}$ .*

Lemma 3 guarantees  $y_n^{crash} < y_{n-1}^{leave} < y_0^{leave}$  and Lemma 2 shows

$$\frac{\partial \hat{V}(r, y)}{\partial r} \begin{cases} > 0 & \text{if } y > y_n^{crash} \\ < 0 & \text{if } y < y_n^{crash} \end{cases} \quad \text{and} \quad \frac{\partial V^+(j, y)}{\partial y} > 0; \frac{\partial^2 V^+(j, y)}{\partial y^2} < 0.$$

If  $y > y_0^{leave}$ , we have  $I = V(0; y_0^{leave}) < V(0; y) < \dots < V(n-1; y)$  such that no depositor withdraws. For  $y \in (y_{n-1}^{leave}, y_0^{leave})$  we have  $V(0; y) < I < V(n-1; y)$  and there is a symmetric mixed strategy equilibrium where each depositor withdraws with probability  $p$ . The expected utility is now given by

$$V(y) = \sum_{j=0}^{n-1} \binom{n-1}{j} p^j (1-p)^{n-1-j} V^+(j, y) \quad (12)$$

and  $V(y) = I$  has to hold in equilibrium. Suppose  $p = 0$ , which yields  $V(y) = V^+(0, y) \stackrel{!}{=} I$  which implies  $y = y_0^{leave}$ . Similarly,  $p = 1$  yields  $V(y) = V^+(n-1, y) \stackrel{!}{=} I$  which implies  $y = y_{n-1}^{leave}$ . By continuity, there is  $p$  with  $0 < p < 1$  and  $V(y) = I$  for all  $y \in (y_{n-1}^{leave}, y_0^{leave})$ . To see that the equilibrium probability  $p$  is strictly monotonically decreasing, suppose that it is not. Let the same probability  $p$  apply to  $y_A$  and  $y_B$  with  $y_{n-1}^{leave} < y_A < y_B < y_0^{leave}$ .

However, since  $V^+(j, y)$  is increasing in  $y$ ,  $V^+(y_A) < V^+(y_B)$  which is a contradiction. Thus,  $p$  must be strictly monotonically decreasing in  $y$ .

Reduce  $y$  to  $y \in (y_n^{crash}; y_{n-1}^{leave})$ . Now, we have  $V(0; y) < \dots < V(n-1; y) < I = W(n-1; y) = W(0; y)$  and all depositors withdraw, but the bank survives. If  $y \in (y_{n-1}^{crash}; y_n^{crash})$  we have  $V(n-1; y_{n-1}^{crash}) = 0 < \lambda IE[x|y_{n-1}^{crash}]$  and still  $V(n-1; y_n^{crash}) < \lambda IE[x|y_n^{crash}] = I$ , such that  $V(n-1; y) < W(n-1; y)$  for all  $y$  and full withdrawal is the equilibrium. Trivially, we get  $V(n-1; y) = W(n-1; y) = \lambda IE[x|y]$  for all  $y < y_{n-1}^{crash}$ .

Step 2: For the sake of completeness, note that additional mixed strategy equilibria may exist for low values of  $y$ ,  $y \in [\tilde{y}_j^{leave}; y_{j+1}^{crash}]$ , where all depositors withdraw with probability  $p$ . Whether these additional equilibria exist, and how many, depends on the distribution of  $y$  and  $x$ . For  $y < y_1^{crash}$  there are always additional equilibria arises beside the full withdrawal equilibrium. Since  $V^+(0, y) = W(0, y) = 0$  and  $\frac{\partial V^+(0, y)}{\partial y} > \frac{\partial W(0, y)}{\partial y}$ , there is always an equilibrium where all depositors leave their money in the bank for all  $y < y_1^{crash}$ . Moreover, given that there are two pure strategies, there is also a mixed strategy equilibrium in this case.

Step 3: Suppose  $D > \hat{D}(\lambda)$  such that  $y_0^{leave} < \tilde{y}_{n-1}^{leave} < y_n^{crash}$ . Note that  $D \leq \bar{x}I$  implies  $\hat{y}_0^{leave} \geq y_1$ , but our assumption  $\lambda > \frac{1}{n}$  yields  $E[x|y_1^{crash}] = \frac{1}{n} \frac{1}{\lambda} < E[x|y_1]$  or  $y_1^{crash} < y_1$ . Thus,  $y_1^{crash} < \hat{y}_0^{leave} = y_0^{leave}$ .

Consider  $j = 0$  first. For all  $y \leq y_1^{crash}$  it holds that  $W(0; y) = \lambda IE[x|y] < V(0; y) = V^+(0; y)$  such that each individual depositor prefers not to withdraw given all others leave their money. For  $y \in (y_1^{crash}; y_0^{leave})$ , we have  $W(0; y) = I > V(0; y) = V^+(0; y)$  such that an individual depositor prefers to withdraw even if no other depositor withdraws. Finally, for all  $y > y_0^{leave}$ , we have  $W(0; y) = I \leq V^+(0; y)$  such that an individual depositor again

prefers not to withdraw in isolation.

Now consider  $j = n - 1$ . For all  $y \leq y_{n-1}^{crash}$  we get  $W(n - 1; y) = \lambda IE[x|y] = V(n - 1; y)$  such that the individual depositor is indifferent, and we assume that he withdraws in this case. For  $y \in (y_{n-1}^{crash}, \tilde{y}_{n-1}^{leave})$ , we get  $W(n - 1; y) = \lambda IE[x|y] > V(n - 1; y)$  such that the individual depositor withdraws if all others do so. For  $y \in (\tilde{y}_{n-1}^{leave}, y_n^{crash})$ , we get  $W(n - 1; y) = \lambda IE[x|y] < V^+(n - 1; y)$  and similarly, for all  $y > y_n^{crash}$  we have  $W(n - 1; y) = I < V^+(n - 1; y)$ . Both cases imply non-withdrawal if all other depositors withdraw.

Taking these results together, we get a continuum of symmetric equilibria of the following structure: All depositors withdraw if  $y \leq \tilde{y}$  while no depositor withdraws if  $y > \tilde{y}$  for all  $\tilde{y} \in [y_0^{leave}, \tilde{y}_{n-1}^{leave}]$ .

Step 4: For the sake of completeness, we again describe the mixed strategy equilibria that may exist for the case where  $D > \hat{D}(\lambda)$  with  $y_0^{leave} < \tilde{y}_{n-1}^{leave} < y_n^{crash}$  for the intermediate region  $[y_0^{leave}, \tilde{y}_{n-1}^{leave}]$ . Suppose each individual depositor withdraws with probability  $p$ . Since  $y < y_n^{crash}$ , for each  $y$ , there is a maximum number  $r$  of depositors the bank is able to pay out. Now we get

$$\begin{aligned} V(y) &= \sum_{j=0}^r \binom{n-1}{j} p^j (1-p)^{n-1-j} V^+(j, y) + \sum_{j=r+1}^{n-1} \binom{n-1}{j} p^j (1-p)^{n-1-j} \lambda E[x|y] I \\ W(y) &= \sum_{j=0}^{r-1} \binom{n-1}{j} p^j (1-p)^{n-1-j} I + \sum_{j=r}^{n-1} \binom{n-1}{j} p^j (1-p)^{n-1-j} \lambda E[x|y] I \end{aligned}$$

For each  $y$ , the withdrawal probability is defined via  $V(y) = W(y)$ . At  $y = y_0^{leave}$ , we get  $p = 0$  as one solution. Note that for all  $y \leq y_{n-1}^{crash}$ ,  $p = 1$  is a solution. For  $y > y_{n-1}^{crash}$  we get  $W(y) < V(y)$  for  $p = 0$  and  $W(y) > V(y)$  for  $p = 1$  such that an interior solution exists by continuity.

### Proof of Lemma 4

1.  $y_n^{crash}$  is defined by  $\lambda E[x|y_n^{crash}] = 1$ . Since  $\lambda < 1$ , we get  $y_1 < y_n^{crash}$ .
2.  $\hat{y}_0^{leave}$  is defined by  $V(0, \hat{y}_0^{leave}) = I$ . Rewrite the definition of  $\hat{y}_0^{leave}$  using  $D = x_0 I$  to get  $\int_0^{x_0} x I f(x|\hat{y}_0^{leave}) dx + \int_{x_0}^{\bar{x}} x_0 I f(x|\hat{y}_0^{leave}) dx = I$ . Eliminating  $I$  on both sides yields  $\int_0^{x_0} x f(x|\hat{y}_0^{leave}) dx + \int_{x_0}^{\bar{x}} x_0 f(x|\hat{y}_0^{leave}) dx = 1 = E[x|y_1] < E[x|\hat{y}_0^{leave}]$ .
3. After no impairment, for each given value of  $y$ , we have  $V^+(j, y) = \int_0^{x_j} \frac{1}{n-j} (nI - \frac{jI}{\lambda E[x|y \geq y_1]}) x f(x|y) dx + D(1 - F(x_j|y))$ . Exchange  $j$  by a continuous variable  $r$  and differentiate to get

$$\frac{\partial V^+(r, y)}{\partial r} = I \frac{n}{(n-r)^2} \left( 1 - \frac{1}{\lambda E[x|y \geq y_1]} \right) \int_0^{x_j} x f(x|y) dx$$

Suppose the bank does not crash after non-impairment even if all depositors withdraw,  $nI - \frac{nI}{\lambda E[x|y \geq y_1]} > 0$  or  $E[x|y \geq y_1] > \frac{1}{\lambda} = E[x|y_n^{crash}]$ . Thus, we have  $\frac{\partial V^+(r, y)}{\partial r} > 0$  for all  $y$  which implies

$$\frac{\partial V(r, NI)}{\partial r} = \int_{y_1}^{\bar{y}} \frac{\partial V^+(r, y)}{\partial r} g(y|y \geq y_1) dy > 0$$

4. Similarly, if the bank crashes after non-impairment if all depositors withdraw, but would be able to bear  $j$  withdrawals at most, we have  $E[x|y \geq y_1] < \frac{1}{\lambda} = E[x|y_n^{crash}]$ . Now, we have  $\frac{\partial V^+(r, y)}{\partial r} < 0$  which implies  $\frac{\partial V(r, NI)}{\partial r} < 0$  for all  $r \leq j$ .

### Proof of Proposition 2

1. Lemma 4 shows that  $y_1 < y_n^{crash}$  and  $y_1 < y_0^{leave}$ . All behavioral boundaries that we know from the Fair Value analysis are above the impairment threshold. Thus, the behavior under HC after an impairment has

occurred is exactly the same as under FV for these low values of  $y$ . Full withdrawal for all  $y \in [\underline{y}; y_1]$  is an equilibrium. (see the discussion on multiple equilibria for low values of  $y$ ; applies here too.)

2. Consider a fairly good project,  $E[x|y \geq y_1] > \frac{1}{\lambda}$  such that  $V(0, NI) < V(1, NI) < \dots < V(n-1, NI)$  after no impairment holds. Since  $\frac{\partial V^+(j, y)}{\partial D} > 0$  we have  $\frac{\partial V(j, NI)}{\partial D} > 0$ . Consider  $V(0; NI)$  first. If  $D = I$ , this implies  $x_0 = 1$  such that

$$\begin{aligned} V(0; NI) &= \int_{y_1}^{\bar{y}} \left( I \int_0^{x_0} x f(x|y) dx + D \int_{x_0}^{\bar{x}} f(x|y) dx \right) g(y|y \geq y_1) dy \\ &< \int_{y_1}^{\bar{y}} \left( I \int_0^1 1 f(x|y) dx + I \int_1^{\bar{x}} f(x|y) dx \right) g(y|y \geq y_1) dy \\ &= \int_{y_1}^{\bar{y}} I g(y|y \geq y_1) dy = I \end{aligned}$$

For  $D = \bar{x}I$  we get  $x_0 = \bar{x}$  and

$$\begin{aligned} V(0; NI) &= \int_{y_1}^{\bar{y}} \left( I \int_0^{x_0} x f(x|y) dx + D \int_{x_0}^{\bar{x}} f(x|y) dx \right) g(y|y \geq y_1) dy \\ &= \int_{y_1}^{\bar{y}} \left( I \int_0^{\bar{x}} x f(x|y) dx \right) g(y|y \geq y_1) dy \\ &= \int_{y_1}^{\bar{y}} I \cdot E[x|y] \cdot g(y|y \geq y_1) dy > I \end{aligned}$$

Since we have  $V(0; NI) < I$  for  $D = I$  and  $V(0; NI) > I$  for  $D = \bar{x}I$ , by continuity there exists  $D_0 \in (I; \bar{x}I)$  such that  $V(0; NI) = I$ . For all  $D > D_0$ , we have  $V(0; NI) > I$  such that all depositors prefer not to withdraw, given that no other depositor withdraws.

Next, consider  $V(n-1; NI)$ . If  $D = I$  we have

$$x_{n-1} \left( nI - \frac{(n-1)I}{\lambda E[x|y \geq y_1]} \right) = D = I$$

such that

$$\begin{aligned}
& V(n-1; NI) \\
&= \int_{y_1}^{\bar{y}} \left( \int_0^{x_{n-1}} \underbrace{\left( nI - \frac{(n-1)I}{\lambda E[x|y \geq y_1]} \right)}_{< I} x f(x|y) dx + I \int_{x_{n-1}}^{\bar{x}} f(x|y) dx \right) g(y|y \geq y_1) dy \\
&< \int_{y_1}^{\bar{y}} \left( \int_0^{x_{n-1}} I f(x|y) dx + I \int_{x_{n-1}}^{\bar{x}} f(x|y) dx \right) g(y|y \geq y_1) dy \\
&= \int_{y_1}^{\bar{y}} I g(y|y \geq y_1) dy = I
\end{aligned}$$

For  $D = \bar{x}I$ , we get

$$\begin{aligned}
V(n-1; NI) &= \int_{y_1}^{\bar{y}} \left( I \int_0^{x_{n-1}} x f(x|y) dx + \bar{x}I \int_{x_{n-1}}^{\bar{x}} f(x|y) dx \right. \\
&\quad \left. + (n-1)I \left( 1 - \frac{1}{\lambda E[x|y \geq y_1]} \right) \int_0^{x_j} x f(x|y) dx \right) g(y|y \geq y_1) dy \\
&= I \left[ \int_{y_1}^{\bar{y}} \underbrace{\left( 1 + (n-1) \left( 1 - \frac{1}{\lambda E[x|y \geq y_1]} \right) \right)}_{> 1} \int_0^{x_j} x f(x|y) dx \right. \\
&\quad \left. + \bar{x} \int_{x_{n-1}}^{\bar{x}} f(x|y) dx \right] g(y|y \geq y_1) dy \\
&> I \int_{y_1}^{\bar{y}} E[x|y] g(y|y \geq y_1) dy > I
\end{aligned}$$

Since  $V(n-1; NI) < I$  for  $D = I$  and  $V(n-1; NI) > I$  for  $D = I\bar{x}$ , by continuity there exists  $D_{n-1} \in (I; \bar{x}I)$  such that  $V(n-1; NI) = I$  at  $D_{n-1}$  and we have  $V(n-1; NI) > I$  for all  $D > D_{n-1}$ . Since  $V(j; NI)$  is increasing in  $D$  and  $j$ ,  $D_{n-1} < D_0(\lambda)$  holds.

Finally, suppose  $D \in (D_{n-1}; D_0(\lambda))$ . Now we have  $V(0; NI) < I$  which means that an individual depositor withdraws given no other depositor

withdraws. Moreover, since  $V(n - 1; NI) > I$ , an individual depositor prefers not to withdraw given all others do so. Since  $V(j; NI)$  is increasing in  $j$ , there is a mixed strategy equilibrium where each depositor withdraws with some probability  $p$ ,  $0 < p < 1$ .

3. Now consider a weak project,  $E[x|y \geq y_1] < \frac{1}{\lambda}$  such that  $V(0, NI) > V(1, NI) > \dots > V(j, NI)$  holds where  $j$  maximum number of depositors the bank is able to pay out, or the smallest integer above  $\tilde{j} = \frac{\lambda E[x|y \geq y_1]}{n}$ . Start with  $V(0; NI)$  again. As step 2 of the proof does not use the assumption on the project type, the same arguments apply<sup>10</sup>. There exists  $D_0$  such that for all  $D > D_0$ ,  $V(0; NI) > I$  holds and no depositor withdraws. Similarly, define  $D_{n-1}$ . Since  $V(0; NI) > V(j, NI)$  we now have  $D_j > D_0$ . Thus, there exist multiple equilibria for some range of parameters.
4. Suppose the project is even weaker,  $E[x|y \geq y_1] < \frac{1}{\lambda} \frac{n-1}{n}$  such that the bank is not able to pay out  $(n - 1)$  depositors after non-impairment. Given  $(n - 1)$  other depositors withdraw, the bank crashes anyway, independent of depositor  $n$ 's decision. Thus, he weakly prefers to withdraw too. Note that this equilibrium does not at all depend on  $D$  as the bank never survives date 1. For all values of  $D$ , there is always an equilibrium where all depositors withdraw immediately.
5. Since for weak projects  $V(j; NI)$  is decreasing in  $j$  and increasing in  $D$ , additional mixed strategy equilibria exist.

For the sake of completeness, we additionally prove the following asymmetric

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<sup>10</sup>Note that by our assumption  $\lambda > 1/n$ , the bank is always able to pay out at least one depositor after a non-impairment. Therefore, we have to compare  $V(0; NI)$  and  $I$  again here.

equilibrium property: *If  $E[x|y \geq y_1] > \frac{1}{n} + \frac{n-1}{n\lambda}$  the bank can prevent full withdrawals by offering a sufficiently large  $D$ .*

To prevent full withdrawal, we need to establish  $V(n-1; NI) > I$ . Plug in the largest possible value,  $D = I\bar{x}$  to get

$$\begin{aligned} V(n-1; NI) &= \int_{y_1}^{\bar{y}} \int_0^{\bar{x}} \left( nI - \frac{(n-1)I}{\lambda E[x|y \geq y_1]} \right) x f(x|y) dx g(y|y \geq y_1) dy \\ &= \left( nI - \frac{(n-1)I}{\lambda E[x|y \geq y_1]} \right) E[x|y \geq y_1] \stackrel{!}{\geq} I \end{aligned}$$

or  $E[x|y \geq y_1] > \frac{1}{n} + \frac{n-1}{n\lambda}$ .

### Proof of Lemma 5

If all depositors withdraw after non-impairment, the participation constraint would reads as

$$\int_{\underline{y}}^{y_1} \lambda I E[x|y] g(y) dy + \int_{y_1}^{y_n^{crash}} \lambda I E[x|y] g(y) dy + \int_{y_n^{crash}}^{\bar{y}} I g(y) dy \geq I.$$

Nevertheless, since  $\lambda E[x|y] < 1$  for all  $y < y_n^{crash}$ , this constraint is never satisfied. Similarly, participation constraint under the mixed strategy cannot be satisfied as the depositors cannot expect more than  $I$  after non-impairment, but receives strictly less than  $I$  after impairment.

If depositors leave the money in the bank after non-impairment, the participation constraint is given as

$$\int_{\underline{y}}^{y_1} \lambda I E[x|y] g(y) dy + \int_{y_1}^{\bar{y}} V^+(0; y) g(y) dy \geq I.$$

For this behavior to be rational, the incentive constraint

$$\int_{y_1}^{\bar{y}} V^+(0; y) g(y|y \geq y_1) dy \geq I$$

has to hold.<sup>11</sup> Rewrite this condition using  $g(y|y \geq y_1) = \frac{g(y)}{1-G(y_1)}$  to get

$$\int_{y_1}^{\bar{y}} V^+(0; y)g(y)dy \geq I(1 - G(y_1))$$

or  $\int_{\underline{y}}^{y_1} Ig(y)dy + \int_{y_1}^{\bar{y}} V^+(0; y)g(y)dy \geq I.$

Compare this to the participation constraint. If the participation constraint holds, the incentive compatibility constraint is automatically satisfied since  $\lambda E[x|y] < 1$  for all  $y < y_1 < y_n^{crash}$ .

The bank's full problem is therefore given by

$$\max_D \int_{y_1}^{\bar{y}} \int_{x_0}^{\bar{x}} n(xI - D) f(x|y)dx g(y)dy$$

subject to the participation constraint. As the bank's expected profit is decreasing in  $D$ , the participation constraint holds with equality.

### Proof of Proposition 3

Plug the binding participation constraint of the depositor into the objective function of the bank. We now get

$$\begin{aligned} & \int_{y_1}^{\bar{y}} n \int_{x_0}^{\bar{x}} (xI - D) f(x|y)dx g(y)dy \\ &= \int_{y_1}^{\bar{y}} n \int_{x_0}^{\bar{x}} xI f(x|y)dx g(y)dy - nI + \int_{\underline{y}}^{y_1} n\lambda IE[x|y]g(y)dy + \int_{y_1}^{\bar{y}} n \int_0^{x_0} Ix f(x|y)dx g(y)dy \\ &= nI(E[x] - 1) - \int_{\underline{y}}^{y_1} nIE[x|y](1 - \lambda)g(y)dy. \end{aligned}$$

Define the critical  $\lambda^{HC}$  such that the bank's profit is equal to zero. We have

$$\lambda^{HC} = 1 - \frac{E[x] - 1}{E[x|y \leq y_1]G(y_1)}, \quad (13)$$

<sup>11</sup>Since  $y_1^{crash} < y_1$ , one depositor can always withdraw the full amount.

while the bank earns positive profits for all  $\lambda > \lambda^{HC}$ .

### Proof of Lemma 6

Suppose  $D > \hat{D}(\lambda)$  such that  $y_1 < y_0^{leave} < y_{n-1}^{leave} < y_n^{crash}$  with  $\tilde{y} \in (y_0^{leave}, y_{n-1}^{leave})$ . Incentive constraints hold by construction. For all  $y < \tilde{y}$ , depositors strictly prefer to withdraw since  $y < \tilde{y} < y_{n-1}^{leave}$  while depositors strictly prefer to leave the money in the bank for all  $y > \tilde{y}$  since  $y > y_0^{leave}$ . Thus, the depositors' participation constraints are

$$\int_{\underline{y}}^{\tilde{y}} \lambda IE[x|y]g(y)dy + \int_{\tilde{y}}^{\bar{y}} V^+(0; y)g(y)dy \geq I.$$

Maximizing the bank's profit,

$$\int_{\tilde{y}}^{\bar{y}} n \int_{x_0}^{\bar{x}} (Ix - D) f(x|y)dxg(y)dy$$

again implies that  $D_{FV}$  is chosen such that the participation constraint is binding. It is easy to verify that  $D_{FV}$  is decreasing in  $\lambda$ . Thus, the presumed subcase  $D_{FV} > \hat{D}(\lambda)$  applies for  $\lambda$  small.

Suppose  $D < \hat{D}(\lambda)$  such that  $y_1 < y_n^{crash} < y_{n-1}^{leave} < y_0^{leave}$  holds. Now, we have three behavioral regions to consider. The respective incentive constraints are satisfied by construction. The participation constraints now read as

$$\int_{\underline{y}}^{y_n^{crash}} \lambda IE[x|y]g(y)dy + \int_{y_n^{crash}}^{y_0^{leave}} Ig(y)dy + \int_{y_0^{leave}}^{\bar{y}} V^+(0; y)g(y)dy \geq I.$$

Since the value on the left hand side of this condition is increasing in  $D$  and the bank's profit function is decreasing in  $D$ , the participation constraint is binding in equilibrium. We started the case with the presumption that  $D$  is small. Thus, this translates to  $\lambda$  being large.

#### Proof of Proposition 4

Solve the binding participation constraint for  $D$ , plug this into the bank's profit function, set profit equal to zero and solve for the minimum value of  $\lambda^{FV}$ . We get

$$\lambda^{FV} = 1 - \frac{E[x] - 1}{E[x|y \leq \tilde{y}]G(\tilde{y})}. \quad (14)$$

#### Proof of Proposition 5

Suppose  $\lambda$  is large, which means  $D$  is small. Under Historic cost, in the limit where  $\lambda \rightarrow 1$ , the binding participation constraint is given by

$$\int_{\underline{y}}^{y_1} IE[x|y]g(y)dy + \int_{y_1}^{\bar{y}} V^+(0; y)g(y)dy = I$$

while under Fair value for  $\lambda \rightarrow 1$  we have  $y_n^{crash} = y_1$  and the participation constraint is

$$\int_{\underline{y}}^{y_1} IE[x|y]g(y)dy + \int_{y_1}^{y_0^{leave}} Ig(y)dy + \int_{y_0^{leave}}^{\bar{y}} V^+(0; y)g(y)dy = I.$$

Suppose the same value of  $D$  applies. Using the equality of the right hand sides, we get

$$\int_{y_1}^{y_0^{leave}} V^+(0; y)g(y)dy \stackrel{!}{=} \int_{y_1}^{y_0^{leave}} Ig(y)dy$$

which is a contraction. Thus, we need to have  $D_{HC} > D_{FV}$ .

Suppose  $\lambda$  is small, which means  $D$  is large;  $D > \hat{D}(\lambda)$ . The participation constraint under Fair value is

$$\int_{\underline{y}}^{\tilde{y}} \lambda IE[x|y]g(y)dy + \int_{\tilde{y}}^{\bar{y}} V^+(0; y)g(y)dy = I$$

where  $\tilde{y} \in (y_0^{leave}; y_{n-1}^{leave})$ . Under HC, we have

$$\int_{\underline{y}}^{y_1} \lambda IE[x|y]g(y)dy + \int_{y_1}^{\bar{y}} V^+(0; y)g(y)dy = I.$$

Rewrite

$$\begin{aligned}
I &= \int_{\underline{y}}^{\tilde{y}} \lambda IE[x|y]g(y)dy + \int_{\tilde{y}}^{\bar{y}} V^+(0; y)g(y)dy \\
&= \int_{\underline{y}}^{y_1} \lambda IE[x|y]g(y)dy + \int_{y_1}^{\tilde{y}} \lambda IE[x|y]g(y)dy + \int_{\tilde{y}}^{\bar{y}} V^+(0; y)g(y)dy \\
&< \int_{\underline{y}}^{y_1} \lambda IE[x|y]g(y)dy + \int_{y_1}^{\bar{y}} V^+(0; y)g(y)dy
\end{aligned}$$

since  $\lambda IE[x|y] < V^+(0; y)$  for  $y \in (y_1; \tilde{y})$ . Thus, for both constraints to be binding, we need to have  $D_{HC} < D_{FV}$ .

### Proof of Proposition 6

Start with  $\lambda$  small. The equilibrium pattern is the same under FV and HC except that the cutoff value is  $y_1$  or  $\tilde{y} > y_1$ . Rewrite the profit functions by plugging in the participation constraint,

$$\Pi^{FV} = nI(E[x] - 1) - \int_{\underline{y}}^{\tilde{y}} nIE[x|y](1 - \lambda)g(y)dy$$

and

$$\Pi^{HC} = nI(E[x] - 1) - \int_{\underline{y}}^{y_1} nIE[x|y](1 - \lambda)g(y)dy.$$

$\Pi^{HC} > \Pi^{FV}$  directly follows from  $y_1 < \tilde{y}$ .

Next, consider  $\lambda$  large which means  $D_{HC} > D_{FV}$ . Compare

$$\Pi^{HC} = \int_{y_1}^{\bar{y}} \int_{x_0}^{\bar{x}} n(Ix - D_{HC})f(x|y)dxg(y)dy$$

and

$$\begin{aligned}
\Pi^{FV} &= \int_{y_n^{crash}}^{y_{n-1}^{leave}} \int_0^{\bar{x}} \left( nI - \frac{nI}{\lambda E[x|y]} \right) x f(x|y) dx g(y) dy \\
&+ \int_{y_{n-1}^{leave}}^{y_0^{leave}} \sum_{j=0}^n \binom{j}{n} p^j (1-p)^{n-j} \int_{x_j}^{\bar{x}} \left[ \left( nI - \frac{jI}{\lambda E[x|y]} \right) x - (n-j) D_{FV} \right] x f(x|y) dx g(y) dy \\
&+ \int_{y_0^{leave}}^{\bar{y}} \int_{x_0}^{\bar{x}} n (Ix - D_{FV}) f(x|y) dx g(y) dy
\end{aligned}$$

Consider the limit of  $\lambda = 1$  which means  $y_n^{crash} = y_1$ . Compare piecewise. For  $y > y_0^{leave}$  the profit under FV is strictly larger since  $D_{FV} < D_{HC}$ . Consider  $(y_1; y_{n-1}^{leave})$  next. Since  $D > I$  we get

$$\begin{aligned}
\int_{y_1}^{y_{n-1}^{leave}} \int_{x_0}^{\bar{x}} n (Ix - D) f(x|y) dx g(y) dy &< \int_{y_1}^{y_{n-1}^{leave}} nI (E[x|y] - 1) g(y) dy \\
&= \int_{y_n^{crash}}^{y_{n-1}^{leave}} \int_0^{\bar{x}} \left( nI - \frac{nI}{\lambda E[x|y]} \right) x f(x|y) dx g(y) dy.
\end{aligned}$$

The intuition is straightforward. Since depositors can only withdraw  $I$ , but not  $D$ , there is more left to the bank under FV.

Similarly arguments apply to the mixed strategy region. Depositors can either withdraw  $I$  (but the bank survives this), or they wait and the bank later on pays out only  $D_{FV} < D_{HC}$ . The probability weighted average of what remains to the bank is therefore larger than what remains to the bank that has to pay out  $D_{HC}$  whenever possible under HC.

$$\begin{aligned}
&\int_{y_{n-1}^{leave}}^{y_0^{leave}} \int_{x_0}^{\bar{x}} n (Ix - D_{HC}) f(x|y) dx g(y) dy \\
< &\int_{y_{n-1}^{leave}}^{y_0^{leave}} \sum_{j=0}^n \binom{j}{n} p^j (1-p)^{n-j} \int_{x_j}^{\bar{x}} \left[ \left( nI - \frac{jI}{\lambda E[x|y]} \right) x - (n-j) D_{FV} \right] x f(x|y) dx g(y) dy
\end{aligned}$$

As all parts are larger, we have  $\Pi^{HC} < \Pi^{FV}$  if  $\lambda$  is sufficiently large.

**Proof of Corollary 1**

Use equations (13) and (14). Since  $y_1 < \tilde{y}$  we have  $\lambda^{FV} > \lambda^{HC}$ . For all  $\lambda \in [\lambda^{HC}; \lambda^{FV})$ , no solution exists under FV, even though a solution exists under HC.

**Proof of Corollary 2**

Trivial, using Lemma 5 and equation (14).

## References

- [1] Mathieu Bouvard, Pierre Chaigneau, and Adolfo de Motta. Transparency in the financial system: Rollover risk and crises. *Journal of Finance*, 70(4):1805–1837, 2015.
- [2] Douglas W. Diamond and Philip H. Dybvig. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91:401–419, 1983.
- [3] Pingyng Gao and Xu Jiang. Reporting discretion, market discipline, and panic runs. Working paper, University of Chicago, 2014.
- [4] Frank Gigler, Chandra Kanodia, Haresh Sapra, and Raghu Venugopalan. Accounting conservatism and the efficiency of debt contracts. *Journal of Accounting Research*, 47(3):767–797, 2009.
- [5] Robert F. Goex and Alfred Wagenhofer. Optimal impairment rules. *Journal of Accounting and Economics*, 48(1):2 – 16, 2009.
- [6] Itay Goldstein and Yaron Leitner. Stress tests and information disclosure. working paper, Wharton School, Feb. 2015.
- [7] Itay Goldstein and Ady Pauzner. Demand-deposit contracts and the probability of bank runs. *The Journal of Finance*, 60(3):1293–1327, 2005.
- [8] Itay Goldstein and Haresh Sapra. Should banks’ stress test results be disclosed? an analysis of the costs and benefits. *Foundations and Trends in Finance*, 8(1):1–54, 2013.

- [9] Tong Lu, Haresh Sapra, and Ajay Subramanian. Asset substitution, debt overhand, and regulation. working paper, University of Chicago, Jan. 2014.
- [10] Paul R. Milgrom. Good news and bad news: Representation theorems and applications. *The Bell Journal of Economics*, 12(2):pp. 380–391, 1981.
- [11] Stephen Morris and Hyung Song Shin. Rethinking multiple equilibria in macroeconomic modelling. *NBER Macroeconomics Annual*, 15:139–182, 2000.
- [12] Guillaume Plantin, Haresh Sapra, and Hyun Song Shin. Marking-to-market: Panacea or pandora’s box? *Journal of Accounting Research*, 46(2):435–460, 2008.
- [13] Edward Simpson Prescott. Should bank supervisors disclose information about their banks? *Economic Quarterly*, 94(1):1–16, 2008.